1. When a sound wave travels through a medium, the particles of that medium oscillate parallel or perpendicular to the velocity of the wave? A

Parallel

2. When a light wave travels through a medium, the electromagnetic field oscillates parallel or perpendicular to the velocity of the wave? B

Perpendicular

3. Suppose you play middle C on the piano (f = 440Hz). If the speed of sound in air at sea level is 330m/s, what is the note’s wavelength? E



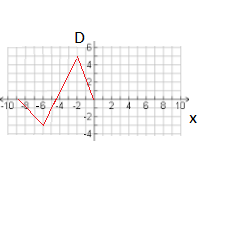
**Example**

Calculate the speed of sound for different T’s and different mmolar’s.

**Example**

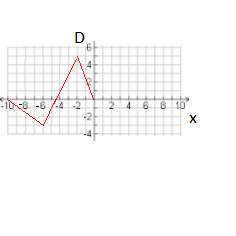
Suppose you’re pulling on rope (like at gyms) with certain tension T. You shake it with frequency f = whatever. And suppose wavelength is whatever. What is speed of waves? What is mass density of rope?

**Question 1.** The wave shown below is progressing to the right at a speed of 3m/s. What will be the velocity of the point at x = 0 at t = 1s? Will it be moving up or down?



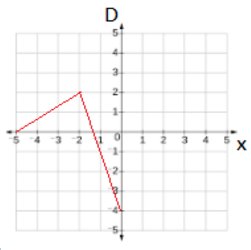
After 1s, the graph will be shifted 3 units to the right, at which point the middle segment of the graph will be at on top of the point x = 0. The slope of this segment is 2unit/s and the velocity of the point will be dD/dt = D΄(x-vt)∙(-v) = (slope)(-v) = (2)(-3) = -6 units/s.

**Question 5.** The wave shown below is progressing to the right at a speed of 2m/s. What will be the velocity of the point at x = 0 at t = 4s? Will it be moving up or down?



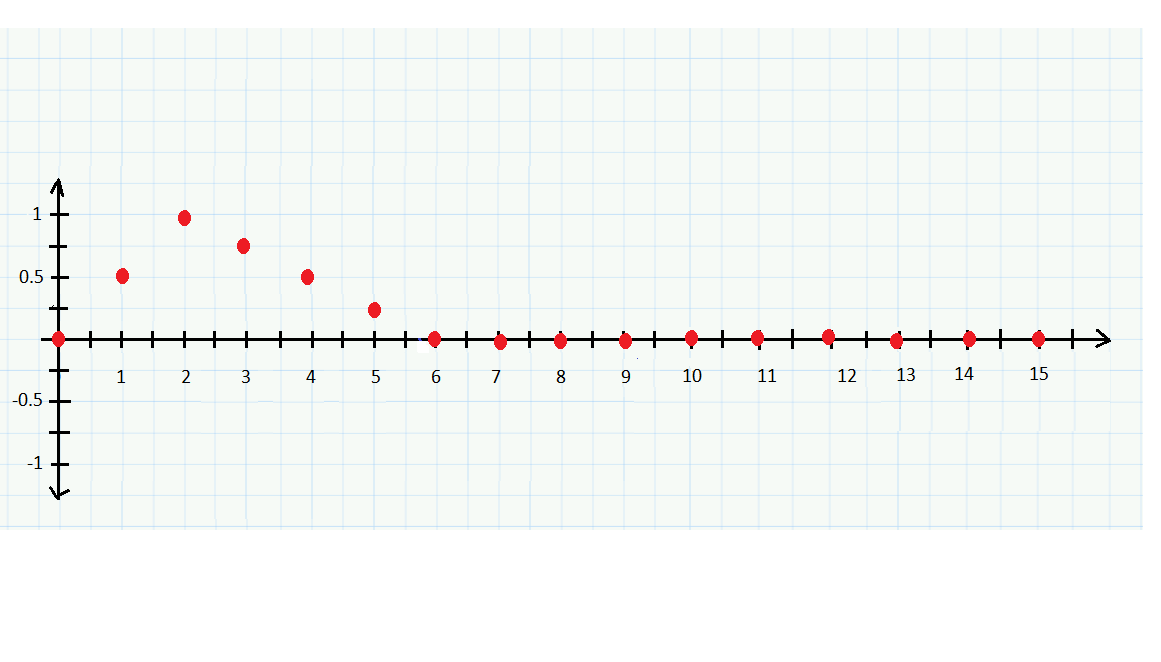
Velocity is v(x,t) = slope(x,t)∙(-vwave) = (-3/4)(-2) = 1.5m/s. And it will be moving up.

**Question 3.** An (admittedly tiny) seismic disturbance, plotted below, is progressing to the right with a speed of 3m/s. What is the displacement and velocity of the point at x = 2m when t = 1s?



Translating graph over 3m, we see that the displacement of point x = 2 would be -1m, and that its velocity would be: v = dD/dt = fˊ(x-vt)∙-v = slope∙-v = -(-3)∙(3m/s) = 9m/s.

**Problem 2.** Rummaging around in the physics department, looking for a demonstration; you find a spring. The spring has a mass m = 2kg, unstretched length ℓ = 10m, and spring constant 6N/m. Now you stretch the spring 5m (you’ve got really long arms). Then you give one end of the spring the following vertical displacement:



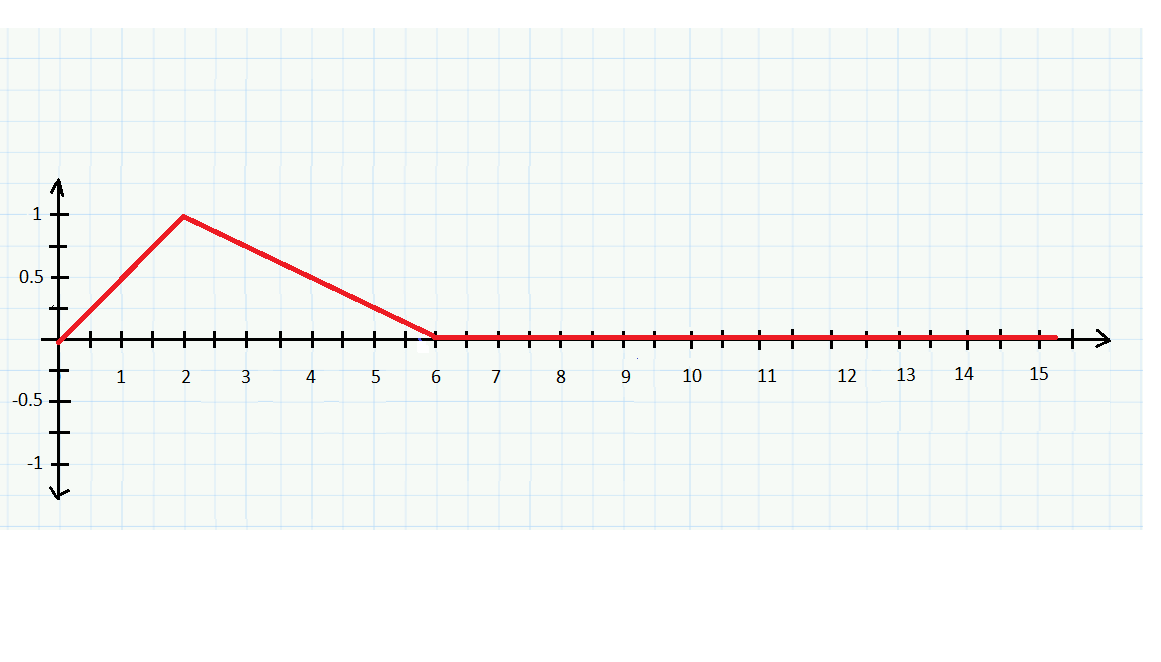
(a) What is the velocity of this wave?

This is:



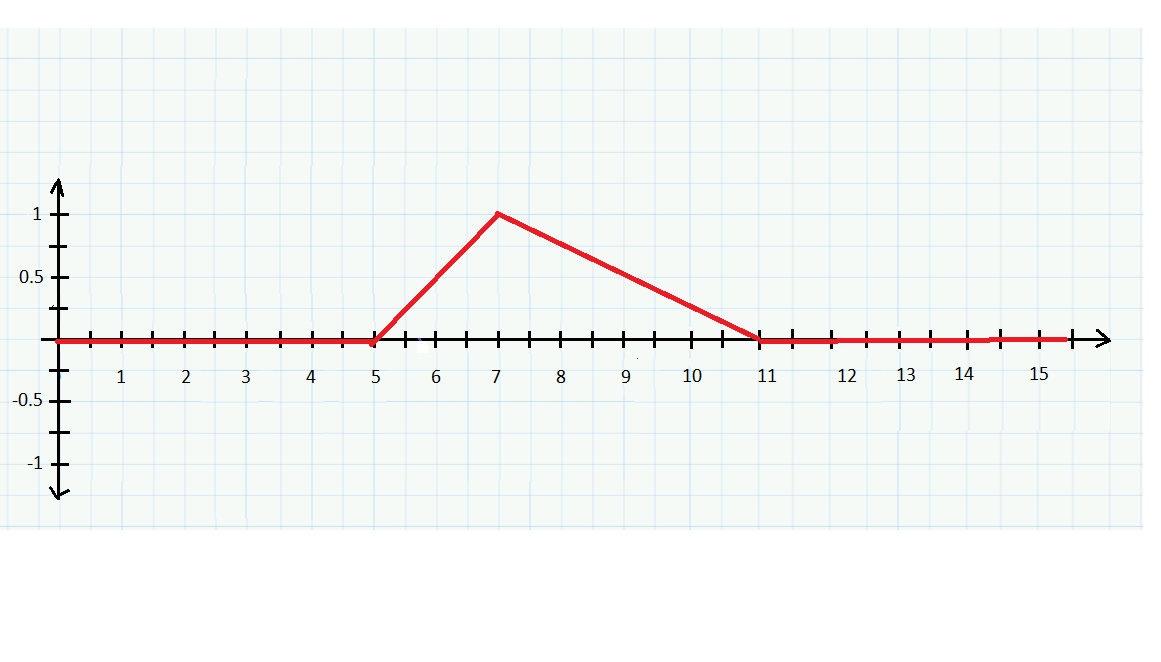
(b) Draw the initial Displacement graph D(x,0):

It’s,

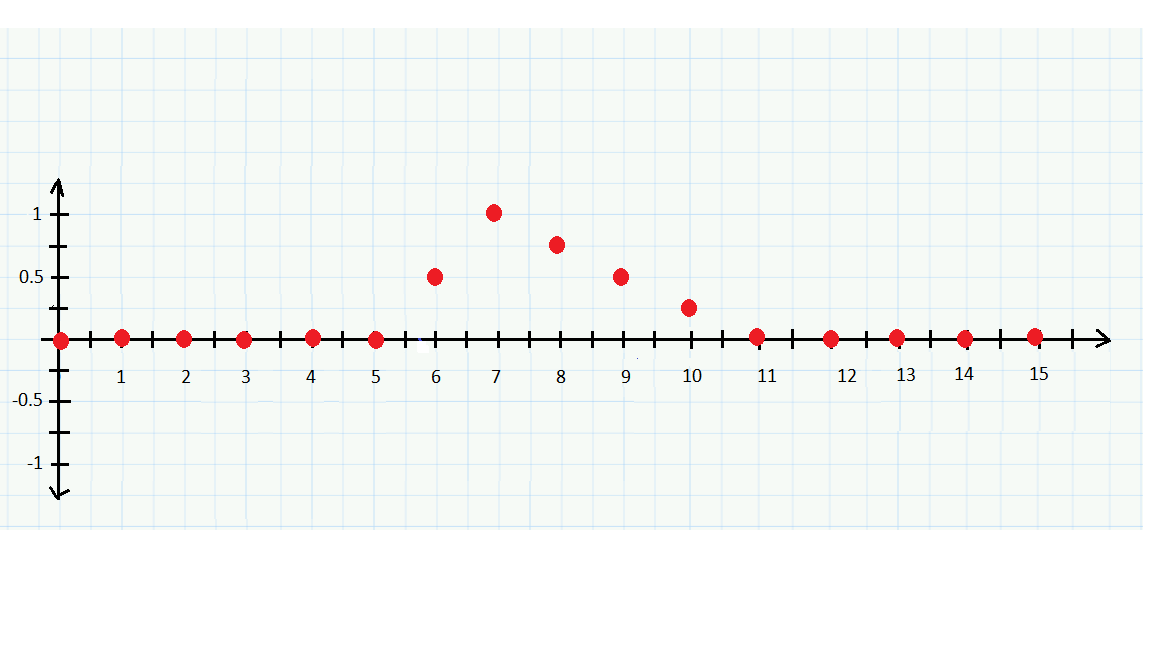


(c) Draw the Displacement graph at t = 0.33s. And also draw what the medium will look like:

And so the wave will have displaced d = vt = 15∙0.33 = 5m. So it’ll look like this:



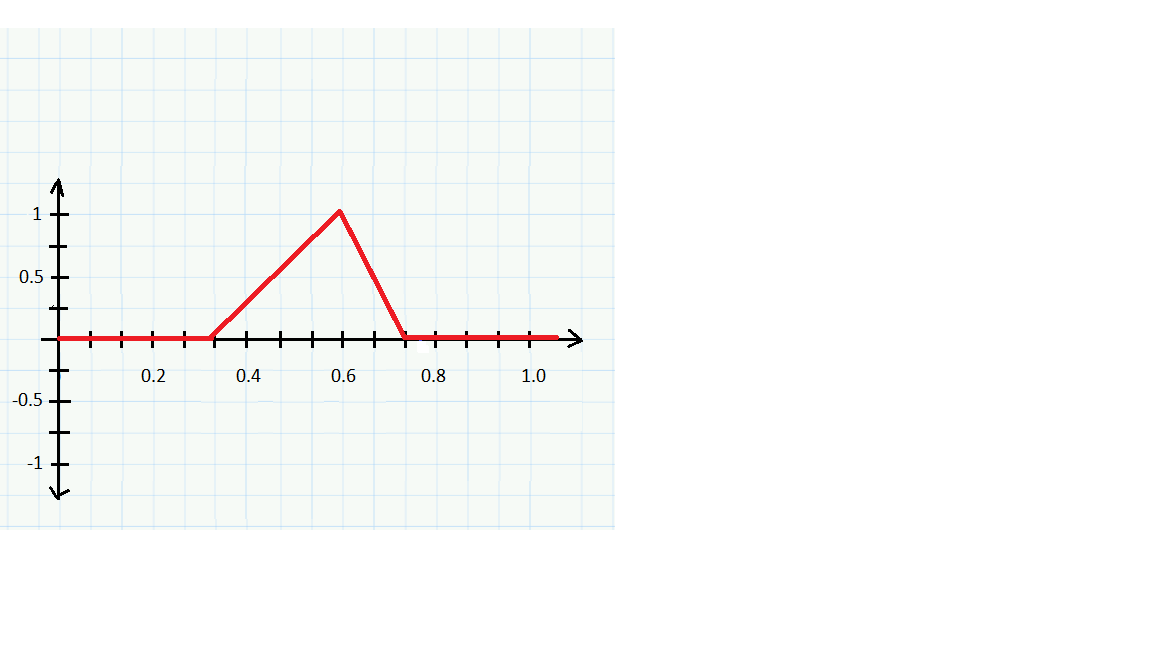
And the medium will reflect this shape:



(d) What will be the displacement and velocity (direction included) of the x = 6m particle at this time?

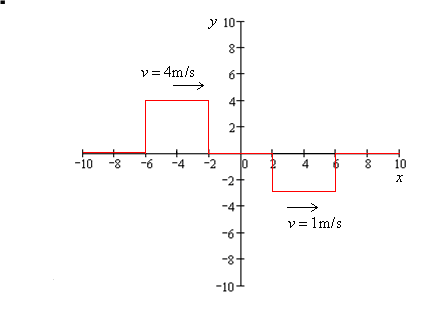
Displacement is apparently D(6,0.33) = 0.5m. And velocity would be: vy = -slope∙vwave = -(1/2)∙15m/s = -7.5m/s. negative sign indicates downward velocity.

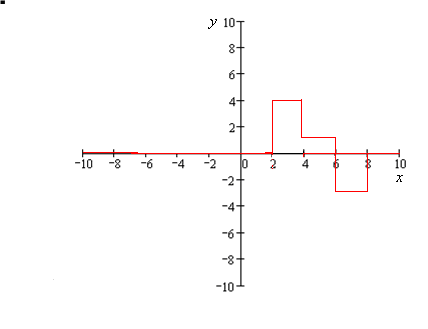
(e) Draw a history graph of the point x = 11m. Between what times is it moving up? Between what times is it moving down? It’s moving up between t = (0.33s, 0.6s), and down between t = (0.6s,0.73s)



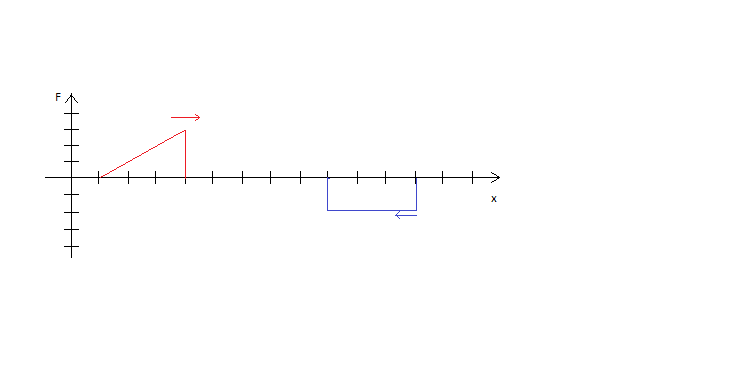
**Example**

9. Consider two wave pulses traveling towards each other on a string. One has a height of 4m, and the other of 3m. If the one on the left is traveling east with a speed of 4m/s, and the one on the right is traveling east with a speed of 1m/s, what will the string look like after 2s? (never mind the fact that wave pulses on a string can travel at only one velocity).

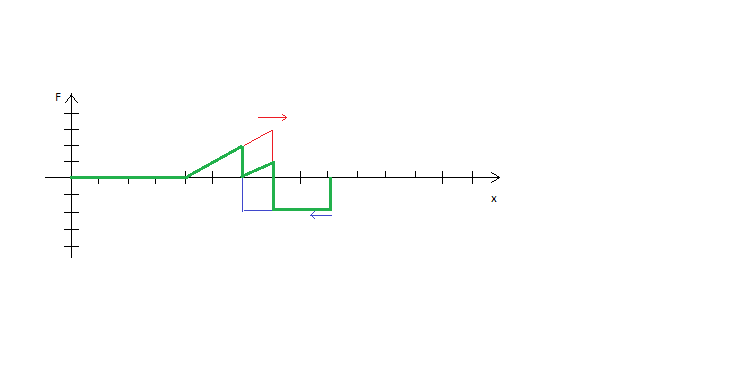




3. The Emperor and Yoda both send force pulses towards each other, as shown below. What will the net force look like in 3s? Each pulse travels at a speed of 1 unit/s.



It will look like, in bold green,



12. If the speed of the wave were 120m/s, and the mass density of the string is μ = 100g/m, what is the tension in the string? E

The tension is given by:



**Example**

A λ = 3m wave is emitted from a speaker. If it travels 10m to someone’s ear, what is its phase once it gets there?

The phase of the wave is:



**Example**

Suppose two people play tug-of-war with a 10m long rope which has a mass of 1.5kg. Suppose they each pull with a force of 500N. If one of them yanks one end of the rope, how long will it take the wave pulse to travel to the other person? How long will it take to come back?

The velocity of the pulse would be:



and so it would take:



for the pulse to reach the other end. Upon being reflected, it will take the same amount of time to come back, for a total travel time of 0.34s.

**Example**

What is the speed of sound in air? water?

B for air is 1.42×105 Pa, and its density is 1.29 kg/m3. So the speed of sound ought to be:



In water the speed of sound is:



**Example**

Suppose survivors of a shipwreck are floating in the water about 400m from the ship, when suddenly it explodes (perhaps because of a torpedo). It is possible to hear the explosion twice – once from the sound waves traveling through the air, and once from the sound waves traveling through the water. What is the time lag between hearing these two explosions?

The sound waves traveling through the air would take a time of:



to reach your ear, while waves traveling through the water would take a time



which would be a lag of 0.93s, which would certainly be noticable. As such, it might give the impression that there were two separate explosions instead of just one.

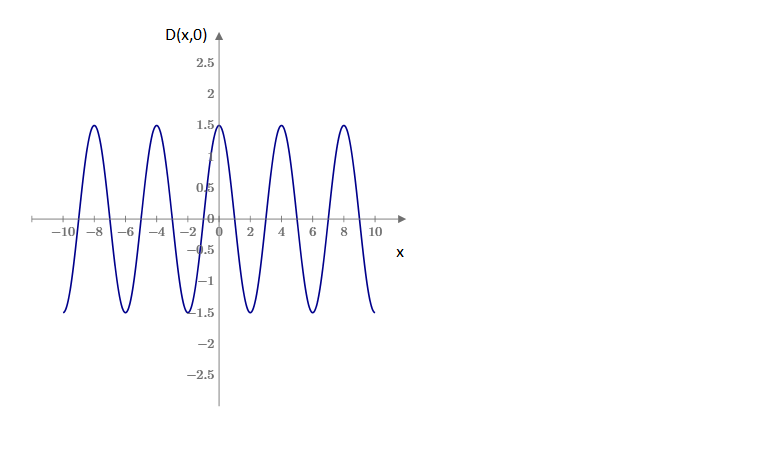
**Sinusoidal Waves**

4. You’re on a boat about 150m from the shore, when a wave passes by. You observe that you bob up and down a distance of about 20cm from the surface of the water and approximately 3 times in 10s. Also you observe that it takes about 25s for the wave to hit the shore after it passes by your boat. Ignoring the phase constant, write down an expression for the displacement of the water as a function of time.

So your amplitude is about A = 0.20m. The frequency is f = 3/10. And the velocity of the wave is v = 150/25 = 6m/s. We can thus infer the wavelength is λ = v/f = 20m. So the formula for the displacement is:



**Question 2.** A sinusoidal wave is traveling to the right with frequency 10Hz. What is equation of wave – in form D(x,t) = Asin(kx – ωt + φ0)? What is max velocity of any point in the medium?



D(x,t) = Asin(kx – ωt + φ0). And A = 1.5, k = 2π/λ = 2π/4 = π/2, ω = 2π/T = 2πf = 2π(10) = 20π, and φ0 = Δx0∙2π/λ = 1∙2π/4 = π/2. So the wave equation is: D(x,t) = 1.5sin(π/2∙x - 20π∙t + π/2). The velocity of a point is dD(x,t)/dt = -20π∙(1.5)cos(π/2∙x - 20πt + π/2). The maximum speed will be the amplitude of this trig function and so vmax = 30π unit/s.

10. Consider a sinusoidal string wave given by the curve . What is the amplitude, wavelength, frequency, and speed of the wave?



2. The equation of a wave on a string is: *y* = (5.3 mm) sin[(25 m-1)*x* + (320 s-1)*t*]  
The tension in the string is 25 N. **(a)** What is the wave speed? **(b)** Find the linear density of this string in grams per meter.

The wave speed is v = ω/k = 320/25 = 12.8 m/s.

The mass density follows from: v = sqrt(T/µ) → v2 = T/µ → µ = T/v2 = 25/(12.82) = 0.153 kg/m.

**Question 4**. A string with linear density 3g/m is stretched along the positive x-axis with tension 27N. One end of the string, at x = 0m, is tied to a hook which oscillates with frequency 60Hz, and maximum displacement 6cm. At t = 0, the hook is at its lowest point. What is the equation for wave that propagates through the medium, in the form D(x,t) = Asin(kx-ωt+φ0)?

Equation is D(x,t) = Asin(kx-ωt+φ0). v = √(T/μ) = √(27/0.003) = 95m/s. A = 0.06m. λ = v/f = 95/60 = 1.58 🡪 k = 2π/1.58 = 4. ω = 2πf = 378. And the phase constant would be -π/2. So D(x,t) = 0.06sin(4x-378t-π/2).

3. A wave with amplitude A = 2cm, and frequency f = 260Hz travels down a string with mass density µ = 5g/m, under tension 100N (typical for a guitar string). What is its speed? What is the equation describing the displacement of a point, x, on the string, as a function of time (can take initial phase φ = 0)? What is the fastest speed that a piece on the string attains?

Well first, the speed of the wave is v = (T/µ)1/2. So,



So the wavelength is:



Therefore the equation of the motion of the point on the string is:

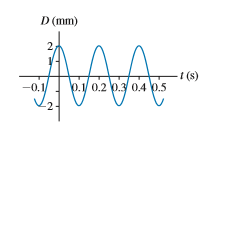


The speed of a particular point, x, on the string is:



In the last line, don't forget to take derivative of inside the cos[ ]. OK, so velocity vy is just an oscillatory function with amplitude 32.8 m/s. And this is the maximum speed therefore of the point on the string (any point).

3. A sound wave pases through the air, traveling to the right at v = 345m/s. The history graph of an affected air molecule at x = 0 is shown below. Write an equation, D(x,t), for the displacement of the wave.



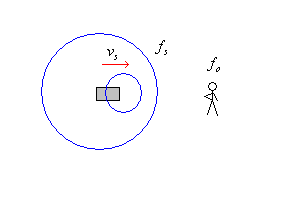
So the graph above is D(0,t) = Acos(k(0)-ωt+φ0) = Acos(-ωt + φ0) = Acos(ωt – φ0). Now A = 2mm. ω = 2π/T = 2π rad/0.2s = 10π rad/s. And the phase constant φ0 would be equal to 0. Also, since ω/k = v → k = ω/v = 10π/345 = 0.029. So the equation for the wave would be:



**Doppler Effect**

**Example**

Suppose an ambulance is coming towards you at a speed of 30m/s blasting its siren at a frequency of 400Hz. What frequency do you hear?



We should expect the frequency to be higher than fs = 400Hz. Plugging in the numbers we get,



which is indeed higher as expected.

**Example**

What frequency will you hear when it is receding from you?

We will expect a frequency lower than 400Hz. Plugging in the numbers,



which is indeed lower. Note that we made the speed of sound, v, negative in this case because the sound waves that would be reaching you would be going to the left in this case.

**Question 3**. You’re driving in a car at 10m/s as an ambulance approaches you. You hear its siren blaring at a Doppler shifted high frequency, and then as it recedes from you, you hear it at a Doppler shifted low frequency.  If the frequency difference is Δf = 70Hz, and speed of the ambulance is 25m/s, what is the frequency of the siren?  Take the speed of sound to be 343m/s.

Imagining the ambulance to be coming towards you from behind (on the left), then on the way towards you, the frequency heard would be f1 = [343-10]/[343 – (25)]fs = 1.047fs. And as it recedes from you, the frequency heard would be f2 = [-343-10]/[-343-(25)]fs = 0.959fs. So then Δf = 1.05fs – 0.96fs = 0.09fs 🡪 fs = Δf/0.09 = 70/0.09 = 795Hz.

If you imagine rather that the ambulance is heading towards you (from the right) then we have: f1 = (-343-10)/(-343-(-25))∙fs = 1.11fs. And then as it recedes from you we have: f2 = (343-10)/(343-(-25))fs = 0.905fs. And so then Δf = 1.11fs – 0.905fs = 0.2fs → fs = Δf/5 = 70/0.2 = 341Hz.

**Question 6**. An ambulance is headed towards you. You hear its siren blaring at a Doppler shifted high frequency, and then as it recedes from you, you hear it at a Doppler shifted low frequency.  If the frequency difference is Δf = 70Hz, and speed of the ambulance is 25m/s, what is the frequency of the siren?  Take the speed of sound to be 343m/s, and assume you are stationary.

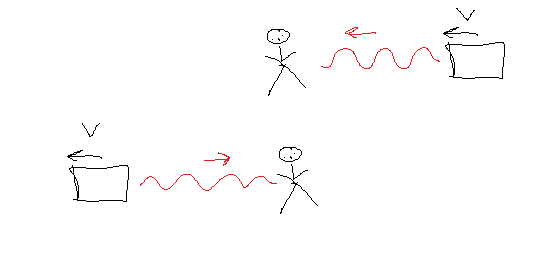
Imagining the ambulance to be coming towards you on the right, then on the way towards you, the frequency heard would be f1 = [-343-0]/[-343 – (-25)]fs = 1.08fs. And as it recedes from you, the frequency heard would be f2 = [343-0]/[343-(-25)]fs = 0.93fs. So then Δf = 1.08fs – 0.93fs = 0.15fs 🡪 fs = Δf/0.15 = 70/0.15 = 467Hz.

**Example**

An opera singer wants to hit 1046 Hz, but can only make it to 1035 Hz. How fast must she walk towards her audience so that they observe 1046?



5. An ambulance is headed towards you. You hear its siren blaring at a Doppler shifted high frequency, and then as it recedes from you, you hear it at a Doppler shifted low frequency. If the frequency difference is Δf = 50Hz, and speed of the ambulance is 20m/s, what is the frequency of the siren? Take the speed of sound to be 331m/s.



The frequency you’d hear as its headed towards you is:



and the frequency you’d hear as its receding away from you is:

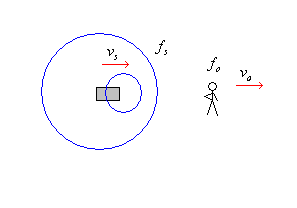


Subtracting the two we get:



**Example**

Suppose the ambulance is approaching you from the left and you are running away from it with a speed of 5m/s. What frequency will you hear? What frequency do you hear when it passes you?



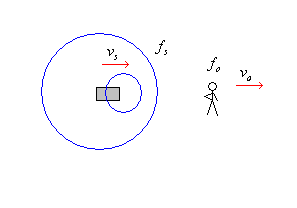


When it passes you, the frequency will be:



**Example**

Suppose you are driving along the road at a speed of 20m/s. An ambulance approaches from behind blaring its siren at a frequency you hear to be 450Hz, and when it passes you hear the frequency to drop to 350Hz. What is the speed of the ambulance?



First for the approaching situation, we have,



when the ambulance is receding, after passing you, the equation would be,



We can solve for vs most directly if we simply divide the two equations into eachother.



**Question 4.** A stationary professional Dopplertician measures the speed of a pitched baseball by aiming his Doppler gun towards it as it approaches. The gun emits a sound wave with a frequency of 900Hz. If the speed of the ball registers as 35m/s, what is the frequency of the signal the gun detects? You may take the speed of sound to be 345m/s.

In general,



and so for the wave emitted by the gun, heading towards the baseball we have:



For the wave on the way back, we have



3. An ambulance with a siren emitting a whine at 1500 Hz overtakes and passes a cyclist pedaling a bike at 2.50 m/s. After being passed, the cyclist hears a frequency of 1400 Hz. How fast is the ambulance moving? (Take the speed of sound in air to be 331 m/s.)

When the ambulance passes the cyclist we have:



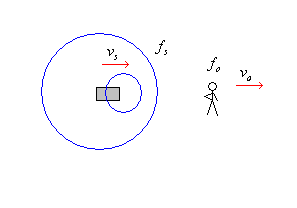
If one were to interpret the problem as cyclist first hears 1500 and then 1400, the answer would be:



Dividing the two equations we get:



??



When it is approaching you we have:



When it passes you, the frequency will be:



note v of sound is negative b/c the sound waves are traveling to the left (to get to you) when the ambulance has passed you.

**Question 7.** You’re driving in trafic. Your speed is 30m/s, and cars in the opposite lane are coming towards you at an unknown speed, vc. You aim your Doppler gun towards traffic, firing a signal at 900Hz, and you receive a signal back at 1250Hz. What is the speed of these cars? You can take the speed of sound to be 345m/s.

In general,



and so for the wave emitted by the gun, heading towards the cars we have:

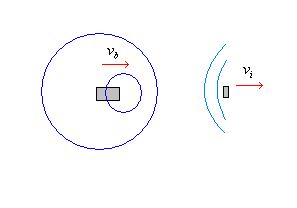


For the wave on the way back, we have



**Example**

Suppose you’re a bat locating an insect. You’re traveling to the right at a speed of 7m/s. You send out an ultrasonic ping of frequency 5000Hz. It comes back 0.06s later, at a frequency of 4890Hz. How far away from you is the insect and what is its velocity? We’ll assume for the sake of discussion that the insect and you flying along a straight line.



First, since the sound wave reflects back in 0.6s, the insect is a distance



away. The speed of the insect can be calculated as follows. First the frequency of the wave intercepted by the insect will be:



These waves will be reflected back to the bat. But now, fi will act as the source frequency and the 4890Hz will be the observed frequency. So we have,

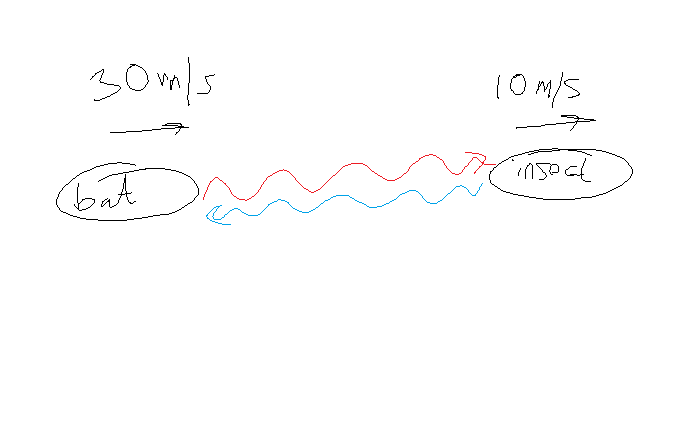


Combining these equations we have,



So the insect is flying to the right (b/c vi is positive) with a speed of 10.8m/s.

**Question 5.** A bat flying at 30m/s emits a sound pulse at f = 1.2 kHz, which reflects off of an insect traveling at 10m/s away it. What frequency will the bat hear? You can take the speed of sound to be 331m/s.



So the insect will intercept a frequency of:



and so the frequency the bat will intercept will be:



**Example**

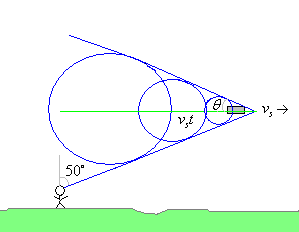
An F-22 flies overhead at Mach 2.2. What angle does the shock cone make with the plane?

The angle would be given by,



**Example**

During a flight show, a plane passes directly overhead with a speed vs. You hear the sonic boom 3s later, when it makes an angle with your position of 50 degrees w/r to the vertical. What is the plane’s altitude and speed?



The speed of the plane is given by:



and the altitude of the plane, h, can be determined from a little trig.



7. A supersonic airplane flying at an altitude *h* passes by overhead. If you hear it 2s later (i.e. because the shock cone just passed your hear) when it makes an angle of 40˚ w/r to you from the vertical, how high is it flying? Take the speed of sound to be 331m/s.

The speed of the plane is:



and the height is therefore,



7. A submarine traveling 10m/s to the right emits a sonar pulse with frequency 80Hz. And it receives the pulse 12s later at a frequency of 82Hz. How far away is the other sub it has detected? What is its velocity? The velocity of sound in water is v = 1484m/s.

The distance to the sub is d = vt = 1482∙12s = 17.8km. Its speed is governed by:



and on the way back,



Putting these equations together we get:



**Sound Intensity**

**Question 6.** A cop listening to you blast Miley Cyrus on your stereo hears a volume of 40dB at a distance of 100m away from your car. What is the volume at a radius 2m away?

So we got:



and so the intensity at 2m is:



And so the volume there is:



**Question**

Suppose volume of one speaker is 10dB, and volume of another is 20dB, then what is volume of both?

6. If I recall correctly from watching *Dune* on TV a million years ago, Paul Atreides possesses a sonic weapon. Suppose that he creates a sonic pulse with power P = 5kW. At what distance from this pulse would the sound intensity be equal to the maximum safe threshold of 120dB?

We have that energy, and therefore power, is conserved. So the intensity of sound a distance r from the source would be:



and we want this intensity to be 120dB, meaning we want:



5. The sound intensity of a tornado siren is I = 0.20 W/m2 at a distance of 30m from the source. At what distance will the intensity be I = 1μW/m2?

Comparing the two intensities we have:





**Question 3.** A speaker playing the The Wall outputs a sound with a power of 75W. At what distance from this speaker will the volume of the sound be 100dB?

We have β = 10log(I/I0) = 10log(P/AI0) = 10log(P/4πr2I0). So we can solve for P… (β/10) = log(P/4πr2I0) →10β/10 = P/4πr2I0 → 4πr2 = (P/I0)10-β/10 → r2 = (P/4πI0)10-β/10 → r = √(P/4πI0)∙10-β/20. And so then r = √(75/4π∙10-12)∙10-100/20 = 24.4m

**Question 6.** You’re playing Dark Side of the Moon at a volume of 80dB, 3m away from the speakers. How far from the speakers will the volume be 60dB?



But since I2A2 = I1A1 → I2/I1 = A1/A2 = 4πr12 / 4πr22 = r12/r22 we can write this as:



So filling in #’s and solving for r2,



**Question 5**. Listening to your favorite song can warm your heart. But how much? Suppose you’re at a concert listening to ‘Call Me Maybe’ at a volume of 85dB. If your heart is roughly 4cm×5cm, and assuming for the sake of discussion that all the sound wave energy hitting your chest there gets absorbed by your heart, how much energy will it have absorbed in 3min?

Energy absorbed would be U = PΔt = IAΔt. And the intensity can be found from the volume equation β = 10log(I/I0) 🡪 I = I010β/10 = (10-12)∙1085/10 = 3.16∙10-4 W/m2. So then U = (3.16∙10-4)(0.04∙0.05)(3∙60) = 1.14∙10-4 J.

**Waves in different media**

Two strings are connected together, both under tension T = 80N. The first has mass density μ1 = 10g/m, and the second has mass density μ2 = 50g/m. A wave pulse with amplitude A1 = 6cm and frequency f1 = 30Hz is sent down the first string. What are:

(a) speed of incident wave? √T/μ = √80/0.015 = 89 m/s

(b) wavelength of incident wave? λ = v/f = 73/30 = 2.97m

(c) energy in wavelength of incident wave? E = (1/2)μ(Aω)2λ = 1.9J

(d) amplitude of reflected wave? Ar = n1 - n2 / n1 + n2 = 2.3cm.

(e) is it inverted? yes

(f) speed of reflected wave? 89 m/s

(g) frequency of reflected wave? 30Hz

(h) wavelength of reflected wave? 2.97m

(i) energy in wavelength of reflected wave? E = (1/2)μ(Aω)2λ = 0.28J

(j) amplitude of transmitted wave? A = 2n1 / n1 + n2 = 3.7cm

(k) is it inverted? no

(l) speed of transmitted wave? v = √T/μ = 40m/s

(m) frequency of transmitted wave? f = 30Hz

(n) wavelength of transmitted wave? λ = v/f = 1.33m

(o) energy in wavelength of transmitted wave? E = 1.62J

**Example**

The speed of sound in air is 345m/s, and the speed in water is 1480m/s. If a sound wave incident in air hits the water’s surface, what fraction of the incident sound wave amplitude will the reflected wave amplitude be? What fraction of the incident sound wave amplitude will the transmitted wave have?



and,



what fraction of energy gets reflected?



and,



**Question 7**. You’re trying catch your pet hamster by luring him into a trap. You’ve tied two strings, S1 and S2, together and connected S2 to the stick holding the trap open. Both strings are of equal length, but S2 has twice the mass of S1. When the hamster is in position, you shake S1 to dislodge the stick and close the trap. But what amplitude must you shake S1 with, if the amplitude of the wave hitting the stick must be 3cm?

Well we have At = [2n1/(n1+n2)]Ai = [2/(1+n2/n1)]Ai. Now n1v1 = n2v2 🡪 n2/n1 = v1/v2 🡪 At = [2/(1+v1/v2)]Ai. Now v1 = √(T/μ1) and v2 = √(T/μ2­), so v1/v2 = √(μ2/μ1) = √(m2/m1) = √2. And so we must solve At = [2/(1+√2)]Ai 🡪 At = 0.84Ai 🡪 3cm = 0.84Ai 🡪 Ai = 3cm/0.84 = 3.57cm.

**Standing Waves**

4. A guitar string 50cm long is plucked. When it is, it vibrates predominantly at its fundamental frequency. If the mass density of the guitar string is 4g/m, to what tension must it be tightened to oscillate at the frequency f = 320 Hz?

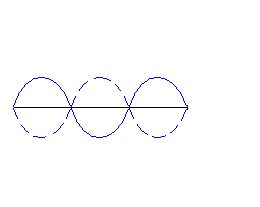
The tension is given by:



Just need v. And v = fλ = f(2L) = (320)(2∙.5) = 320 m/s. So T is:



11. A standing wave on a string looks like shown below. If the length of the string is 2m, and the fibration frequecy of the string is 60Hz, what is the speed of the wave on the string? D



1.5 wavelengths occupy the string. So the wavelength is given by:



and so the speed is:



**Question 1.** A heavy piece of hanging sculpture is suspended by a 90 cm-long, 5.0 g steel wire. When the wind blows hard, the wire hums at its fundamental frequency of 75 Hz. What is the mass of the sculpture?

The fundamental wavelength is given by the resonance condition:



And the frequency is given by f = λ/v = 2L/√(T/μ)



Solving for msculpture we have:



**Example**

Suppose you have a rope 4m long, with a mass density of μ = 0.25kg/m attached to a wall. Holding your end relatively still with a force of 200N, what wavelengths of standing waves can you make on the rope? What frequencies would these waves have? What speed would the waves have?

The wavelengths you can make are the governed by the following relationship.



and so,



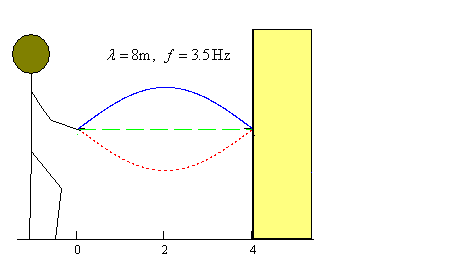
where n is any natural number. The possible frequencies are related to λ by fλ = v, and so:



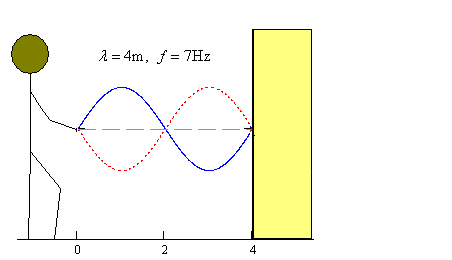
For our situation, v = √T/μ = √200/(0.25) = 28 m/s. And L = 4, so,



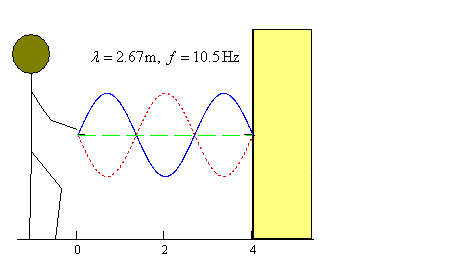
where n can be any natural number: 1, 2, 3, 4, …These are illustrated below. Below we have n = 1. n = 1 corresponds to the so-called fundamental frequency of the string, i.e., the lowest frequency that can be a standing wave.



and n = 2. The n = 2 frequency is called the 1st *harmonic*.



and n = 3. The frequency corresponding to n = 3 is called the 2nd *harmonic*, etc.

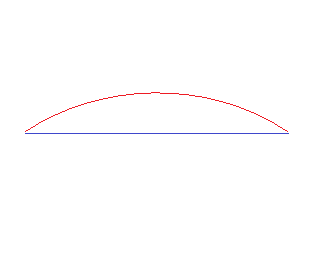


etc. Note how the value of *n* happens to correspond to the number of anti-nodes on the string.

Finally, the speed of the string waves is of course: v = √T/μ = √200/(0.25) = 28 m/s.

1. A guitar string 40cm long is plucked. When it is, it vibrates predominantly at its fundamental frequency. If the mass density of the guitar string is 3g/m, to what tension must it be tightened to oscillate at the frequency f = 260 Hz (approximately middle C)?

The standing wave corresponding to the fundamental frequency looks like this:



and so L = λ/2 → λ = 2L = 0.8m. The frequency will therefore be:



Solving for T we have:



Filling in the numbers...



7. A guitar string must tuned to 330Hz (i.e. that should be its fundamental frequency). If its mass is 0.12mg, and its length is 25cm, what tension should it be tightened to?

Well, we should have



**Question 7.** A violinist places her finger so that a vibrating section of a 1.0g/m string has a length of 30cm, then she draws her bow across it. A listener nearby hears a 550Hz note. What is the tension in the string?

The wavelength of the string will be:



The string is oscillating at f = 550Hz so the wave speed is:



and so the tension in the string is:



**Example**

To what tension do you need to tighten a 40cm long guitar string (mass density μ = 0.005 kg/m) to make its fundamental frequency be middle C, i.e. 262 Hz?

The standing wave frequencies you can make on the guitar string are *fn = nv/2L*. The fundamental frequency is given by n = 1, and so f0 = v/2L. Now v = √(T/μ), and so we must have,



**Question 6.** A guitar string has a mass m = 0.090g, and length ℓ = 75cm. To what tension must it be tightened to produce a fundamental frequency of 440Hz?

Guitar string resonant frequencies are given by Δφ = 2*πm* → (2π/λ)(2L) + 2∙π = 2πm → 2L/λ + 1 = m → λ = 2L/(m-1). The fundamental wavelength is the largest, i.e., one for which m = 2 → λ = 2L. The frequency is f = v/λ = v/2L. And the velocity is v = √(T/μ) so we have f = √(T/μ)∙1/2L. Solving for T we get T = μ(2Lf)2 = (0.00009/0.75)(2∙0.75∙440)2 = 52N

**Question 2.** An old mining tunnel disappears into a hillside. You would like to know how long the tunnel is, but it's too dangerous to go inside. Recalling your recent physics class, you decide to try setting up standing-wave resonances inside the tunnel. Using your subsonic amplifier and loudspeaker, you find resonances at 5.0 Hz and 7.0 Hz, and at no frequencies between these. It's rather chilly inside the tunnel, so you estimate the sound speed to be 337 m/s. What is the length of the tunnel?

Resonance condition gives wavelengths to be:



Associated frequencies are:



The change in f is given by:



And so we have:



**Question**

Why do the guitar strings that play higher notes have a smaller cross section?

Because a smaller cross section, means that μ is smaller for these strings. This being the case, the velocity of the waves they support is higher, because v = √(T/μ). And this being the case, the standing wave frequencies on the string are higher since f = v/λ is inversely proportional to μ.

**Question**

Why do you also pluck shorter strings to make higher notes?

The fundamental frequency is given by f1 = v/2L. And so is inversely proportional to L. Therefore the smaller L is, the higher f1 is. Basically, by shortening the string, you are forcing the wavelength of the standing wave to be smaller, and thus the frequency to be larger.

**Question**

Why do notes sound louder when you increase the amplitude of the plucked string?

The reason is that the loudness of the sound that you hear is related to the power delivered by these waves (the power is delivered to your ear drum which is why the power of the wave is relevant to *you*). If we increase the power, then the sound appears louder. Since P ~ A2, increasing the amplitude will increase the power.

**Example**

Suppose you’re sitting in a bathroom stall, humming to yourself. You note that humming in a particularly deep voice, and gradually increasing the frequency, the stall walls start to resonate (yes this happened to me) when you get to a certain frequency. If the stall walls are about 1.5 high and not attached to the floor or ceiling, what is likely this frequency?

We would approximate the stall walls as an open pipe (since its open at the floor and ceiling). The length of the pipe is 1.5m, and so the possible resonant frequencies (standing wave frequencies) are:



Now the speed of sound is v = √B/ρ ≈ 331m/s. So…



Now, 240 Hz is middle C on a piano, and since this was a deep voice I was humming in, likely, the note was 110 Hz. This corresponds to the note A2 on a piano scale (about 1.5 octaves below middle C).

**Question**

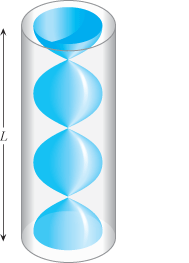
Why are instruments that play higher notes shorter than instruments that play lower notes?

The reason is that when you play a flute for instance, you excite the standing wave frequencies in the flute. Assuming a flute is like and open/open pipe, these frequencies are:



and so the shorter the length of the instrument, L, the higher the frequencies, fn. This is also the purpose of covering the holes in the instrument. By covering the holes, you are basically changing the effective length of the instrument.

3. An organ pipe is open at both ends. It is producing sound at its *n* = 3 harmonic, the frequency of which is 500 Hz. What is the length of the pipe?



The wavelength of the sound wave is λ = v/f = (341)/(500) = 0.68m. But also, looking at the vibration pattern, the wavelength is given by (3/2)λ = L. So L = (3/2)(0.68) = 1.02m.

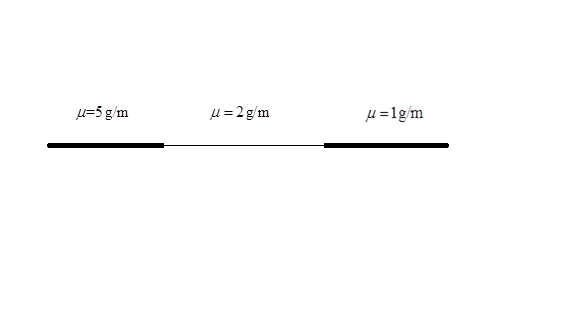
11. Suppose you have 2-liter bottle of coke, and you blow air over the top of it (without the cap on). And suppose that the liquid level is 15cm below the top of the bottle. If you can get the sound to resonant under those conditions, what is the lowest frequency this sound wave could have?

The fundamental wavelength is:



**Transmission and Reflection**

8. You tie three ropes together, with the mass densities indicated. The middle rope has a length ℓ = 3m. List the smallest three frequencies with which you can shake the rope on the left so that the waves that travel down the strings to the right do not reflect. You may suppose that all ropes are under tension T = 90N.



The wave will transmit through if the wave in the middle forms a standing wave. And we will have:



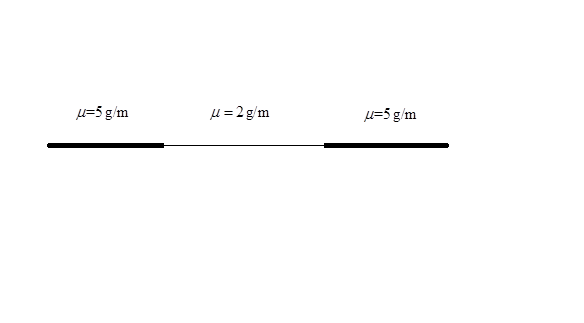
The velocity of such waves will be:



And so the frequency will be:



8. You tie three ropes together, with the mass densities indicated. The middle rope has a length ℓ = 3m. What is the smallest frequency with which you can shake the rope on the left so that the waves that travel down the strings to the right do not reflect? You may suppose that all ropes are under tension T = 90N.



The wave will transmit through if the wave in the middle forms a standing wave. Noting that the media on either side have a larger ‘index of refraction’, we’ll have:



The velocity of such waves will be:

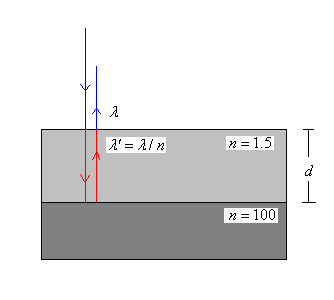


And so the frequency will be:



1. Suppose that the metallic surface of an airplane has an index of refraction of 100. We would like to make the airplane invisible to radar waves of wavelength 3cm. A possible means is to coat the surface of the plane with a polymer (n = 1.50). What minimum thickness would be required to accomplish the job?

The original wave (λ = 3cm) will be incident on the polymer, and will reflect, undergoing a phase shift. Transmitted wave will also under go a phase shift,



Since they both undergo a phase shift, in order for the red guy to destructively interfere with the blue one, we need it to advance by a net λ/2. So we need,

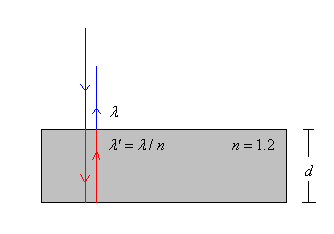


The minimum thickness will be had by taking m = 0. And so,



**Example 1: Soap bubble**

Consider light impinging upon a soap bubble, of thickness d, and n = 1.2. Let’s figure out the minimum thickness required to have a light wave, say λ = 700nm, *constructively* interfere with itself. What is the smallest thickness that will cause destructive interference of the waves? Note that the reflected and refracted rays are drawn different colors for clarity – not because they *are* different colors.



The light we see will be the light that is traveling back up towards our eye. This light will be the superposition of the reflected blue and refracted/reflected red rays going in the **y** direction. When the blue ray hits the n = 1.2 medium, since its coming from air (n = 1), it will shift phase by φblue = π radians. When the red ray enters the medium n = 1.2, first of all its wavelength will shrink to λ′ = λ/n. It will travel a total distance of 2d and so will undergo a phase shift through the process, but will not shift phase upon reflection since it reflects off of a lower index of refraction (air). So the total phase shift of the red ray will be φred = (2π/λ′)(2d) = 4πnd/λ.

If the red ray is to meet the blue one at the upper surface and constructively interfere with it, then they will will have to be out of phase by an even amount of 2π. So we need:



The smallest thickness will be when meven = 0, so…



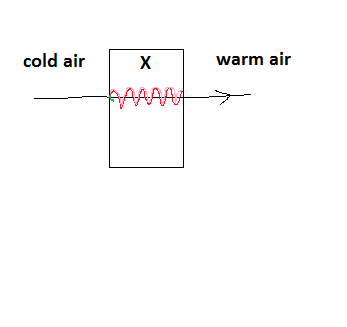
What is the smallest thickness that will cause destructive interference of the light wave? If we want *destructive* interference, then we need Δφ = moddπ. So:



The minimum (non-zero thickness) occurs when m = 1.



7. Suppose you have cold air at temperature T = 250K, and warm air at temperature T = 310K, separated by a wall made of material X. Suppose also the speed of sound in the wall is v = 325m/s. If you project a 220Hz sound wave through this material, what minimum thickness would result in perfect transmission? You can take the molar mass of air to be 29g/mole.



For perfect transmission we need the sound wave to resonate inside the wall. This means that the phase change the wave goes through as it travels through the medium (red), then reflects off the right end and travels back through the medium (pink), and finally reflects off the left and end and rejoins the original wave (green) must be 2πm. To work out the phase change we need to know the velocities of this sound wave in the different media. The velocity of the wave v in cold air/warm air is:



This means that the indices of refraction are ordered as ncold air > nwall > nwarm air, since n is inversely proportional to wave velocity. So when the wave reflects off of warm air (pink) there will be no phase change. When it reflects off of cold air (green) there will be. Also the wavelength of the wave in the wall is λ = v/f = 325m/s / 220Hz = 1.48m. So finally, letting x = the width of the wall, the condition for constructive interference is that:

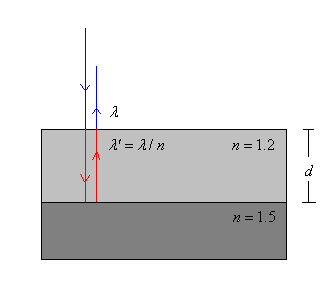


The smallest width will be when m = 1.



**Example 2**

What if we put another material beneath the soap bubble? Consider the following arrangement with the soap bubble having an index of refraction n = 1.2, and another material below having an n of 1.5. We want the minimum non-zero thickness for constructive and destructive interference between the reflected and refracted ray.



In this case, both rays will phase shift by π upon reflection, since both reflect off of a higher n. So the reflected blue ray will phase shift by φblue = π, and the red refracted/reflected ray will phase shift by φred = (2π/λ′)(2d) + π = 4πnd/λ + π. To get constructive interference we need:



the minimum d is therefore when m = 2,



The minimum d for destructive interference is when:



the minimum d is therefore when m = 1,



4. Suppose that the metallic surface of an airplane has an index of refraction of ∞. We would like to make the airplane invisible to radar waves of wavelength 5cm. A possible means is to coat the surface of the plane with a polymer (n = 1.30). What minimum thickness would be required to accomplish the job?

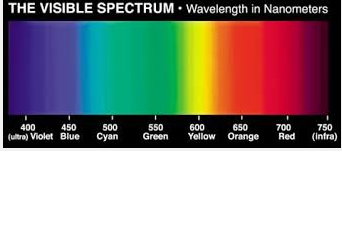
Change in phase of first reflected ray will be φ1 = π. Change in phase of second reflected ray will be:



Subtracting the two, we must have:



**Question 7.** A thin layer (thickness 700nm) of oil (n­oil = 1.5) rests on top of a water (n = 1.33) filled crack in the asphalt. Which visible wavelenth of light would you see looking down, i.e., which visible wavelengths of light would be most strongly reflected?



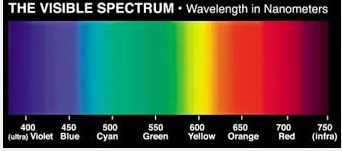


and the wavelengths in the air and oil are related via nλ = n΄λ΄ → (1)λ = (1.5)λ΄. So,



So 600nm, and 467nm are the visible ones.

**Question 8**. After having aced all of your exams you treat yourself to a bubble bath. And happen to you notice light shining through a thin soap bubble membrane with index of refraction n = 1.7 and thickness d = 0.3μm. Expert at physics as you are, you instantly calculate which color of light you will see on the other end by calculating which wavelength in the visible spectrum will not reflect. So what is your result (specify wavelength please….)?



So for the transmitted wave, we’d want it to constructively interfere with itself. There will be 0 phase shifts for the transmitted wave, and so we need:



m = 2 gives the only visible wavelength, λ = 1020nm/2 = 510nm.

**Question 8.** Consider a soap bubble with index of refraction n = 1.7. When you shine visible light on it, you see a green color (λ = 550nm) on the other side at one point. What are the two smallest possible thickness of the soap bubble at that point? Note you’ll want to find the thicknesses that result in strong transmission of that wavelength through the bubble.

We need destructive interference between the two reflected waves. Only the first reflected wave will phase shift upon reflection, and so we’ll have:



Now λ΄ is the wavelength in the bubble, which is related to the wavelength outside via nλ = n΄λ΄ → (1)λ = (1.7)λ΄ → λ΄ = λ/1.7 = 550nm/1.7. So we have:



**Question 8**. What is the thinnest layer of clear plastic (n = 1.5) that would be invisible to light of wavelength λ = 700nm? Assume the light is traveling in air before and after.

So for the transmitted wave, we’d want it to constructively interfere with itself. There will be 0 phase shifts for the transmitted wave, and so we need:



**Question 3.** A soap bubble is essentially a very thin film of water (*n* = 1.33) surrounded by air. The colors that you see in soap bubbles are produced by interference. What visible wavelengths of light (400nm to 700nm) are strongly reflected from a 383-nm-thick soap bubble?

Resonance condition is:



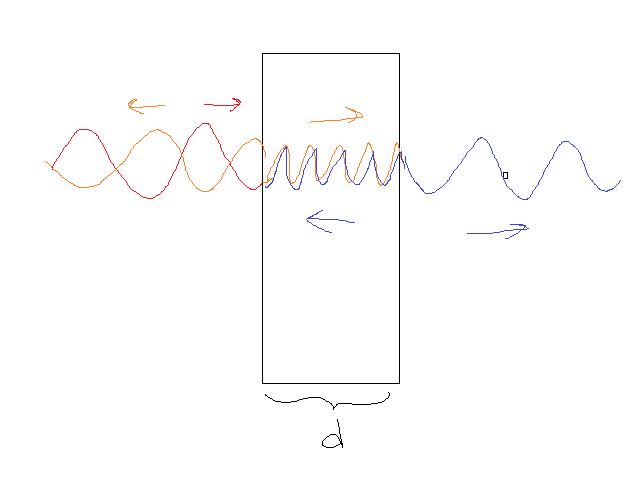
From Snell’s law, we have n1λ1 = n2λ2 🡪 λ1 (wavelength in air) = n2λ2/n1 = (1.33)λ2. So,



So the visible wavelengths are 689nm and 408nm.

8. What is the thinnest layer of clear plastic (n = 1.5) that would be invisible to light of wavelength λ = 600nm? Assume the light is traveling in air before and after.

We have something like the following:



the phase shift of the reflected orange wave is φ1 = π. The phase shift of the reflected blue wave is φ2 = 2kd. And we want the phase difference between these two waves to be 2π(n+1/2) so that they cancel out and allow the wave to go straight through. So we have:



But then recognize that λ΄ (calling it λ2 now) is given by n1λ1 = n2λ2 → λ2 = (n1/n2)λ1 = (1/1.5)(600nm) = 400nm. So filling this in and setting n = 0 we get:



Diffraction

**Example 1**

Suppose we have a diffraction grating for which d = 1μm, and L = 2m. If we shine light of λ = 500nm. At what height above the central maximum will the 2nd dark band appear?

Using the small angle approximation, we can say that the height of the second dark band is given by,



**Example 2**

Suppose we take the double slit apparatus with d = 1μm and shine white light on it. What will be the separation between red and blue light first order maximums?

The blue light diffraction pattern we can expect to be narrower than the red line one since blue light has a smaller wavelength. In any event, the first order maximum for blue light (λ = 400nm) will be:



while the red (λ = 700nm) one will be at,



So the separation between maximas will be 1.4m – 0.8m = 0.6m. So we see that the diffraction grating will separate the colors in white light into a spectrum, kind of like a prism does.

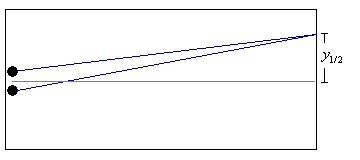
11. Suppose you have two radar dishes on the ground, 200m apart, emitting waves 1m in length. Suppose you’re rocket man (or woman), levitating up in the air a distance of 1000m, directly between the radar dishes. In order to escape detection, you want to fly to a spot where the radar waves cancel. What minimum horizontal distance along the line connecting the dishes should you travel?

If you have microwaves with intensity I = 500 W/m2 incident on a material (n = 1.7) parallel to the normal, what intensity of microwaves will be transmitted inside the material?



**Example 3**

Suppose we have two coherent sound speakers 1.2m apart emitting sound at a frequency of 500Hz (vsound = 343m/s). And suppose you’re standing 15m away from the center of them against the opposite wall. How far along the wall must you walk in order to hit the first ‘dead’ spot.



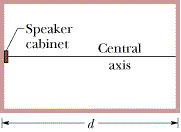
The analysis we’ve been making with light waves applies to all types of waves. The wavelength of the sound waves is:



and so plugging this into our formula,



8. Sound waves with frequency 3000 Hz and speed 343 m/s diffract through the rectangular opening of a speaker cabinet and into a large auditorium of length 100 m. The opening, which has a horizontal width of 35.8 cm, faces a wall 100 m away. Along that wall, how far from the central axis of that wall in meters will a listener be at the first diffraction minimum and thus have difficulty hearing the sound? (Neglect reflections.)



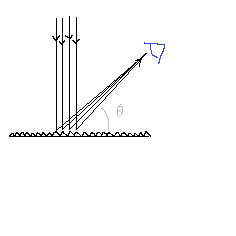
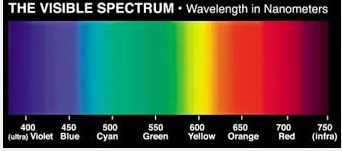
The single slit diffraction equation, for the minimas is:



Now sinθ ≈ tanθ = y/d. So solving for y we have:



9. Suppose you have light reflecting off of a grated surface with grooves approximately 1.4μm apart. Estimate which color you would see at the angle θ = 50° from the horizontal by determining which frequency of light would constructively interfere with itself at that angle.

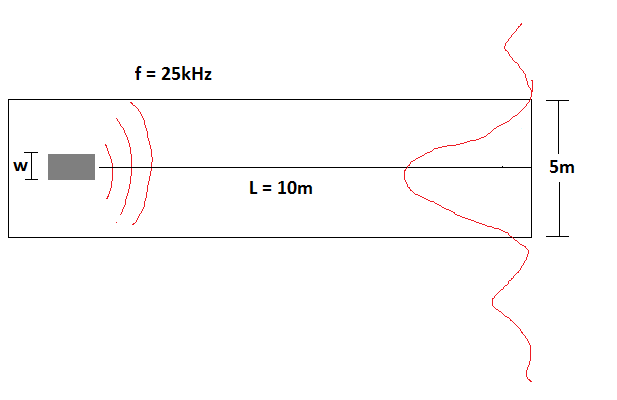


Filling in n = 1, 2, 3, etc. we get:



The only visible wavelength is 450nm, which corresponds to blue. Frequency would be f = c/λ = 3∙108/4.5∙10-7 = 6.7∙1014 Hz.

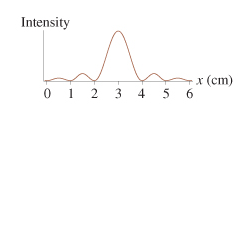
7. You design radar guns for the police. The radar gun is designed to emit sound waves with a frequency 25kHz. What should be the width, w, of the gun to ensure that minimums in the diffraction pattern, from 10m onwards, are located beyond the road’s boundaries? You can take the speed of sound to be v = 345 m/s.



We have:



**Question 9**. The intensity of light that hits a screen in front of an aperature. The aperature is 4m away from the screen, and light with wavelength λ = 600nm is used. Does the aperature have a single slit, or multiple slits. And if a single slit aperature, what is its width; if it’s a multiple slit aperature, what is the spacing between the slits?



It’s a single slit aperature. Distance between the central maximum and first minimum is 1cm. And so the width of the aperature is given by:



**Example 3**

Suppose we shine a laser pointer on a screen. If the light is red (λ = 700nm) and the diameter of the laser beam is 2mm, and we’re shining it on a wall 3m away, what will the diameter of the bright spot on the wall.

The radius is approximately just the distance from the central maximum to the first minimum. This is:



Therefore the diameter would be d = 2y1min = 4.66mm.

**Example 4**

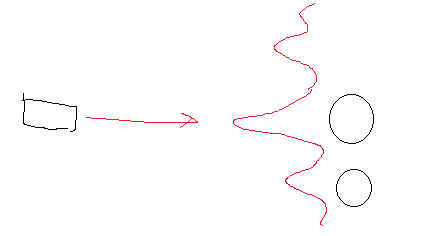
Suppose we string a human hair between two screws, and shine blue light on it, and observe the projection of light on a screen 1m away. If the first minima is located at a distance of 1cm from the central maximum, what is the width of the hair?

Using our equation and solving for *a*, we get,

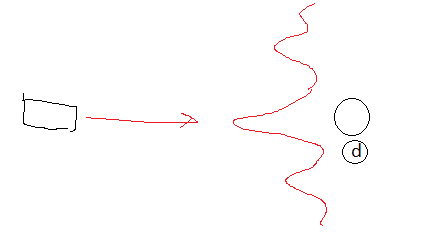


**Assasins**

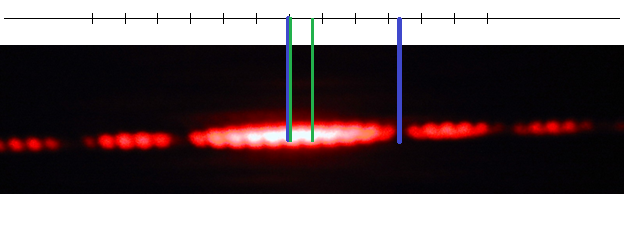
Suppose you want to kill two enemies at once with only a single laser beam λ = 600nm. Enemies are 2m apart and 150m away:



Kill one person, but save dog. Or suppose you want to eliminate one person, but avoid the dog nearby, so that you want dog in the minimum,



**Question 1**. You pass a laser through a double slit diffraction grating, resulting on the pattern seen below. Suppose that λ = 630nm, and the distance between the grating and screen is L = 1.5m. Estimate the slit widths *a*, and slit spacing *d*. You can take the scale on the ruler below to be 1cm. Also, draw vertical lines from the pattern to the ruler so I can see what distances you are estimating.



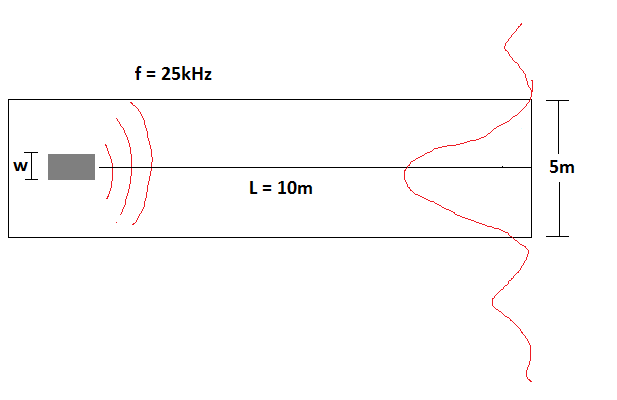
The slit width, a, will be smaller than d and so is responsible for the wide pattern. The distance between the central maximum and first order minimum is illustrated in blue and is about 3.2cm:



The slit spacing, d, is responsible for the small patter. The distance between the central maximum and the next maximum is illustrated in green, and is about 0.8cm. So then d is given by:



7. You design radar guns for the police. The radar gun is designed to emit sound waves with a frequency 25kHz. What should be the width, w, of the gun to ensure that minimums in the diffraction pattern, from 10m onwards, are located beyond the road’s boundaries? You can take the speed of sound to be v = 345 m/s.



We have:



**Problem 8.**

You are part of a special ops unit targeting certain sites for destruction. As such your job is to shine an infrared light on an object from a distance of 50m away. The width of the beam can only be 5cm at most by the time it hits the target. The width of the light at its source is 3cm. What wavelength of light should you use? Note that you can consider the width of the beam when it hits the object to be the distance between the first two minima on either side of the central maximum.

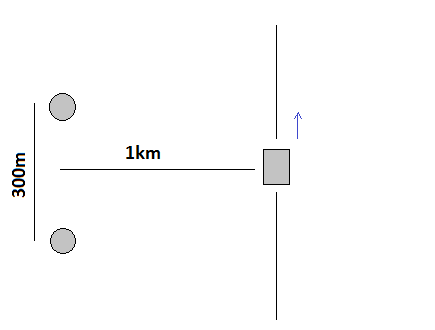
We have:



and we’re looking for distance between two minimums. The angle to the first minimum will be, using geometry, θ = tan-1(0.025/50) = 0.029◦. And then plugging this into the single slit equation:



6. Two radio antennas broadcast the NPR news channel (92.1FM, i.e. f = 92.1×106Hz) a distance of 300m apart. Starting from directly in between the two antennas, how far would you drive north before you hit the first ‘dead’ spot, where the radio signal is weak? You can take the speed of EM radiation to be c = 3×108m/s.



The angle to the first dead spot will be



Also have that tangent of angle is distance you drive, y, divided by 1km. So



9. An x-ray beam of wavelength 98pm undergoes first-order reflection from a crystal when its angle of incidence to a crystal face is 25 °. What is the spacing between lattice planes?

Have the equation:



will accept also,



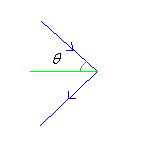
**Question 4**. Light from a helium-neon laser (*λ*=633nm) is incident on a single slit. What is the largest slit width for which there are no minima in the diffraction pattern?

We’d have asinθ = mλ. If no minima then we must have θ = 90 where m = 1 → asin90 = (1)λ → a = λ = 633nm.

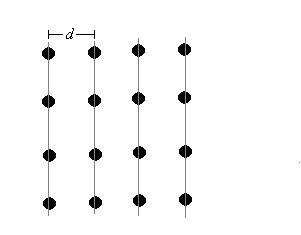
**Crystallography**

**Example 1**

Suppose you send green light (λ = 550nm) into a crystal in the horizontal direction (θ = 0) and gradually increase the angle until you find a bright spot at 20 degrees. What is the set of lattice planes causing this?



Well the direction of the lattice planes would be perpendicular to the bisector of the two angles. Since it is the first resonance encountered, n must be 1. For consider if it were higher, then there would yet be another resonance detectable at the smaller n. So this being the case, we have,



where the distance d is given by (m=2)



**Example 2**

Just note the following observation. If we want to probe the structure of a crystal with lattice planes somewhere around, say, 1nm, then we cannot use visible light because suppose we try the same approach as above. Use the same color light, and see for what angles we’ll get a resonance given d ~ 1nm.



but for every meven, this are undefined. Therefore we get no resonances (bright spots). In order for the cosine function to be defined we need the argument to be less than 1. This means that we need,



Plugging in the various values of meven = 2, 4, 6, …, we see that the largest λ can be is 2d (m = 2). So



So we see that in order to get any results at all, we need to probe the crystal with wavelengths that are no larger than dimensions of the crystal. Since visible light has a wavelength of at least 400nm, the smallest length scale that we can expect to use light to probe is around 400nm = 0.4μm.